NOTE ON PROF. NICHOLSON'S SINGULAR VALUE OF Π.

BY PROF. WILLIAM WOOLSEY JOHNSON.

IF we regard equation (6), p. 150, as simply equivalent to $\infty.0 = 2 \div \pi$ it presents no difficulty; but if, on the other hand, the symbols $(1—1)^{\frac{1}{2}}$ and $(1—1)^{-\frac{1}{2}}$ be supposed to stand for the limits of $(1—x)^{\frac{1}{2}}$ and $(1—x)^{-\frac{1}{2}}$ when $x = 1$, the result appears paradoxical, since then the product of these quantities and therefore its limit is equal to unity.

The former is in fact correct for although equation (2) is the result of putting $x = 1$ in the expansion of $(1-x)^n$, (3) is not, since the process of transforming the infinite series into an infinite product is applieable only when $x = 1$. Thus equation (3) means nothing more than that the infinite product in the second member has zero for its limit; in like manner equation (4) means only that the infinite product in its second member has no limit.

Moreover, the product obtained by taking an infinite number of factors from each series may have any value we choose, for this value is a function of the ratio of the infinite numbers of the factors taken from the two series. If p factors be taken from (4) and q factors from (3) the value of the product, when p and q are both infinite but $p \div q = a$, is

$$\frac{a^n \sin n\pi}{n\pi};$$

putting $n = \frac{1}{2}$ and assuming $a = 1$, as implied in the manner of writing Wallis' Theorem, the result becomes $2 \div \pi$.

NOTE ON EXPERIMENTAL CONFIRMATION OF THEORETICAL DEDUCTION, BY THE EDITOR.—IF a plane surface is ruled with parallel and equidistant lines and a slender rod, the length of which equals the perpendicular distance between two consecutive lines, is thrown at hazard upon the plane, the probability that it will fall across a line is $2 \div \pi$. (See *Mathematical Monthly*, Vol. II, p. 236.)

If we denote this probability by P, we shall have

$$P = \frac{2}{\pi} = \frac{2}{3.14159} = .6366.$$

Hence a rod thrown at hazard upon the plane 10,000 times should fall across a line 6366 times.

At the recent Montreal meeting of the American Association for the Advancement of Science, Prof. Mendenhall exhibited before Section A. of the Association the result of 30,000 experiments which he had performed by

casting a rod upon a ruled surface as above described. And as he claimed that he had avoided all conceivable sources of bias in pitching the rod, it was expected that the mean of so great a number of experiments would agree very nearly with the theoretical value of P; but, contrary to expectation, though the mean of the first 3000 experiments gave almost exactly the theoretical value of P, from that point onward to the end of the 30,000 trials the experimental result steadily diverged from the theoretical value of P.

In the discussion which followed the reading of this paper, various reasons were assigned for the discrepancy between the theoretcal and practical results, none of which seemed satisfactory to the Section. Among other possible sources of error in the result of the trials a personal eq'n was suggested by Prof. Mendenhall. The source of the error in this case, however, will, most likely, be found in that physiological characteristic of muscular action in accordance with which any act, frequently repeated, is continued unconciously in the same way. As this peculiarity cannot be eliminated by a personal equation, it follows that, in experiments of this kind, instead of approaching the theoretical value indefinitely by extending the experiments to a very great number, the liability of falling into special habits after a cert'n number of experiments have been performed not only renders further experimentation, by the same individual. useless, but even prejudicial.

INTEGRATION OF SOME GENERAL CLASSES OF TRIGONOMETRIC FUNCTIONS.

BY PROF. P. H. PHILBRICK, IOWA STATE UNIVERSITY, IOWA CITY.

In solving some of the more difficult problems that naturally arise in the higher branches of Applied Mathematics and in connection with a general course in engineering, one frequently has occasion to perform integrations, the formulas for which are not found in the books, at least not in those accessible to the operator.

Some recent experience of this nature led me to integrate some very general classes of Trigonometric functions to which those I incidentally encountered belong, and I offer the results and the methods of arriving at them for such consideration as they deserve. It is perhaps needless to remark that the subsidiary formulas required, are developed and used without special enquiry in regard to their existence elsewhere.